Kronecker Product-Based Space-Time Block Codes

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***Abstract*—This letter presents a new approach to construct space-time block codes (STBCs) based on the Kronecker product. The proposed Kronecker-structured STBCs (K-STBCs) are specially designed for constant modulus constellations and include the classical linear dispersion (LD) codes as a special case. Its inherent Kronecker structure offers additional coding gains due to the mutual spreading of multiple space-time codewords. At the receiver, an efficient algorithm is derived by recasting the decoding of the K-STBCs as a rank-one tensor approximation problem. The proposed decoding algorithm provides a good bit error ratio (BER) performance especially, in the low signal-to-noise ratio (SNR) regime. In particular, our numerical results show that K-STBCs outperform the existing quasi-orthogonal STBCs due to the denoising gain offered by the proposed rank-one approximation based detector.**

***Index Terms*—Kronecker coding, space time block codes, least squares rank-one approximations, tensor-power method detector.**

I. Introduction

MASSIVE multiple-input-multiple-output (MIMO) is a contender for the next generation of wireless communication systems to achieve extremely high data rates. Most of the previous work on massive-MIMO has focused on channel estimation, precoding, and resource allocation to improve the overall spectral efficiency. Only a little attention has been given to the idea of broadcasting channels, so far, where the base station (BS) needs to transmit common signaling to all users in the cell. This includes also users that are not served, e.g., to tell the inactive users to send pilot signals. Such a short message should be conveyed in a reliable way. While transmitting, the short message could only include one packet over one channel realization, and, therefore, time or frequency diversity might not be possible. In this regard, an extra spatial diversity is desirable using a coded transmission with low complexity decoders. An orthogonal space-time block code (STBC) is proposed in [1] for the BS to broadcast this common information.

STBCs have been employed to increase the capacity in various communication standards. More specifically, there exists a class of complex orthogonal designs (CODs) that provides transmit diversity and a simple maximum-likelihood (ML) decoder [2]. However, the code rate, i.e., the ratio of independent complex symbols to the total number of time slots taken to transmit a COD block, decreases as more transmit antennas are used. To overcome this challenge, quaternion orthogonal designs (QODs) are proposed in [3] to exploit dual-polarized antennas and use polarization as additional transmit diversity. The work [4] jointly exploits space, time, and polarization diversities with decoupled ML decoding. The authors of [5] resorts to random matrix theory to design new STBCs for MIMO systems. A new STBCs is proposed in [6] based on spatial-permutation modulation (SPM) and spatial modulation (SM) which is known as STBCs-SPM. A new upper bound for the *bit error probability* for STBCs-SM is derived in [7] based on Verdu's theorem [8]. The performance of MIMO-STBCs over correlated fading channels is analyzed in [9].

A receiver strategy is proposed in [10], which exploits a cross-coding by using tensor space-time coding (TSTC) based on the Kronecker product. The work [11] proposes a construction of a linear error-correcting tensor code using the Kronecker product property, where the coding structure is defined by parity check matrices based on the discrete Fourier transform (DFT) codes. The authors in [12], introduces a random access scheme, where the transmitted symbols are rank-1 tensors constructed from Grassmannian sub-constellations, the low-rank structure of which helps the separation of users by using a tensor decomposition algorithm. In [13], a sparse Kronecker-product (SKP) coding is used for unsourced multiple access scheme based on encoding data symbols *via* the Kronecker product. A ML folded-tree decoding strategy is proposed in [14] for Reed-Muller (RM) and polar codes using a Kronecker product-based (KPB) method.

To achieve full rate quasi-orthogonal STBCs, a small amount of non-orthogonality is introduced in the STBCs designs, which results in different quasi-orthogonal STBCs schemes like the ABBA code [15]. Another example in the same family is the extended Alamouti (EA) STBCs [16], [17]. Special linear dispersion (LD) codes have been proposed for any combination of transmitting and receiving antennas

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that satisfy an information-theoretic optimality property [18]. In [19], a Kronecker-structure of especially designed constant modulus constellations has been proposed to encode multiple modulated symbols vectors, which is efficiently exploited at the receiver via rank-one approximations schemes. The so-called Kronecker-rank-one detector (Kronecker-RoD) shows an improved performance in the low signal-to-noise ratio (SNR) regime compared to the existing error-correcting schemes. Multi-linear encoding and decoding schemes for multiple-input-multiple-output (MIMO) systems have been studied in [20].

In this letter, we propose a new approach to construct space-time block codes (STBCs) based on the Kronecker product. The proposed Kronecker-structured STBCs (K-STBCs) are especially designed for constant modulus constellations and includes the classical linear dispersion (LD) codes as a special case. We show that the Kronecker product structure of K-STBCs offers additional coding gains due to the mutual spreading of multiple space-time codewords. At the receiver side, we propose an efficient algorithm to decode the K-STBCs based on a rank-one tensor approximation. The proposed K-STBCs show an improved performance especially in the low SNR regime due to the inherent noise rejection capability of the proposed decoder. Our numerical simulations also evaluate the performance of the proposed K-STBC when combined with zero-forcing (ZF) or minimum mean square error (MMSE) equalizers, over both correlated, which is generated by using the geometrical structure of antennas [21] and uncorrelated fading channels. We further compare our proposed scheme with the Gaussian approximation bound for short-length block codes [22].

Notation: Scalars are denoted by lower-case italic letters $(a, b, \ldots)$, vectors by bold lower-case italic letters $(\boldsymbol{a}, \boldsymbol{b}, \ldots)$, matrices by bold upper-case italic letters $(\boldsymbol{A}, \boldsymbol{B}, \ldots)$, tensors are defined by calligraphic upper-case letters $(\mathcal{A}, \mathcal{B}, \ldots)$, $\boldsymbol{A}^{\mathrm{T}}$, $\boldsymbol{A}^{*}$, $\boldsymbol{A}^{\mathrm{H}}$ stand for transpose, conjugate and Hermitian of $\boldsymbol{A}$, respectively. The operators $\otimes$ and $\circ$ define the Kronecker and the outer product, respectively. For an Nth order tensor $\mathcal{Y} \in \mathbb{C}^{L_1 \times \cdots \times L_N}$, the n-mode unfolding of $\mathcal{Y}$ is the matrix $[\mathcal{Y}]_{(n)} = \mathbb{C}^{L_n \times L_1 \ldots L_{n-1} L_{n+1} \ldots L_N}$. $\mathbb{E}[\cdot]$ is the expectation operator and $\boldsymbol{I}$ is the identity matrix.

II. System Model

We consider a multiple-input-single-output (MISO) system with M transmit antennas and a single receive antenna operating over Rayleigh fading channels. The received signal can be modeled by the following input-output relation:

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{h} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{y} \in \mathbb{C}^{N \times 1}$ is the received signal vector, $\boldsymbol{X} \in \mathbb{C}^{N \times M}$ is the space time codeword matrix, where N is the time span of the block code, i.e., the number of time slots associated with the spreading of the data symbols, while $\boldsymbol{h}$ is the channel vector, with $\boldsymbol{h} \sim \mathcal{CN}(\boldsymbol{0}_{M \times 1}, \boldsymbol{I}_M)$ and defined as

$$\boldsymbol{h} = [h_1, \ldots, h_M]^{\mathrm{T}} \in \mathbb{C}^{M \times 1}. \tag{2}$$

Finally, $\boldsymbol{n} \sim \mathcal{CN}(\boldsymbol{0}_{M \times 1}, \sigma_n^2 \boldsymbol{I}_M)$ is the circularly symmetric additive white Gaussian noise vector with variance σ_n^2.

The structure of the STBCs codeword $\boldsymbol{X}$ is detailed as follows. The serial data is converted into multiple parallel data streams modulated with a constant modulus, i.e., μ-ary phase shift keying (PSK), constellation. The K-STBCs encoding consists of two steps. First, the K symbol vectors $\boldsymbol{s}_1 \in \mathbb{C}^{P_1 \times 1}, \ldots, \boldsymbol{s}_K \in \mathbb{C}^{P_K \times 1}$ are individually mapped into K STBCs words $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_K$. Then, the final codeword is generated *via* multiple Kronecker products of the K codewords, leading to

$$\boldsymbol{X} = \boldsymbol{X}_1 \otimes \cdots \otimes \boldsymbol{X}_K \in \mathbb{C}^{N \times M}. \tag{3}$$

This encoding operation implies a mutual spreading of the K codewords into both spatial and temporal domains by means of the Kronecker product. By applying the vec$(\cdot)$ operator on both sides of (3), we obtain

$$\boldsymbol{x} = \mathrm{vec}(\boldsymbol{X}_1 \otimes \cdots \otimes \boldsymbol{X}_K) \in \mathbb{C}^{NM \times 1}, \tag{4}$$

where $\boldsymbol{x} = \mathrm{vec}(\boldsymbol{X}) \in \mathbb{C}^{NM \times 1}$, $\boldsymbol{X}_k \in \mathbb{C}^{n_k \times m_k}$, $N = n_1 \ldots n_K$, $M = m_1 \ldots m_K$, where n_k is the time slots spanned by the kth codeword, and m_k defines the antenna group associated with the spatial spreading of the kth codeword. Recall the Kronecker product property $\mathrm{vec}(\boldsymbol{A} \otimes \boldsymbol{B}) = \boldsymbol{P}(\mathrm{vec}(\boldsymbol{a}) \otimes \mathrm{vec}(\boldsymbol{b}))$, where $\boldsymbol{P} \in \mathbb{R}^{NM \times NM}$ is a block row permutation matrix. Using this property leads to

$$\boldsymbol{x} = \boldsymbol{P}[\boldsymbol{x}_1 \otimes \cdots \otimes \boldsymbol{x}_K]. \tag{5}$$

Now, making use of the LD codes formalism [18] and defining $\boldsymbol{x}_k = \mathrm{vec}(\boldsymbol{X}_k) \in \mathbb{C}^{n_k m_k \times 1}$, we have

$$\boldsymbol{x}_k = \mathrm{vec}\left(\sum_{i_k=1}^{P_k} \boldsymbol{A}_{i_k}^{(k)} s_{i_k}^{(k)}\right). \tag{6}$$

Substituting (6) into (4), we obtain

$$\boldsymbol{x} = \mathrm{vec}\left(\left[\sum_{i_1=1}^{P_1} \boldsymbol{A}_{i_1}^{(1)} s_{i_1}^{(1)}\right] \otimes \cdots \otimes \left[\sum_{i_K=1}^{P_K} \boldsymbol{A}_{i_K}^{(K)} s_{i_K}^{(K)}\right]\right), \tag{7}$$

where $\{\boldsymbol{A}_1^{(k)}, \ldots, \boldsymbol{A}_{P_k}^{(k)}\}$ are the dispersion matrixes associated with the k-th K-STBCs codeword. Noting that $\boldsymbol{a}_{i_k}^{(k)} = \mathrm{vec}(\boldsymbol{A}_{i_k}^{(k)}) \in \mathbb{R}^{n_k m_k \times 1}$, let us define

$$\bar{\boldsymbol{A}}_k = \left[\boldsymbol{a}_{i_1}^{(k)} \ldots \boldsymbol{a}_{P_k}^{(k)}\right] \in \mathbb{R}^{n_k m_k \times P_k}, \tag{8}$$

$$\boldsymbol{s}_k = \left[s_{i_1}^{(k)} \ldots s_{P_k}^{(k)}\right]^{\mathrm{T}} \in \mathbb{C}^{P_k \times 1}, \tag{9}$$

where $\bar{\boldsymbol{A}}_k$ can be seen as an effective LD matrix associated with P_k symbols to generate the kth codeword $\boldsymbol{x}_k$. Since, $\bar{\boldsymbol{A}}_k$ as given in (8) is a collection of P_k vectorized dispersion matrices. It is known that the design of a LD code crucially depends on the choice of the associated dispersion matrices [18]. Herein, we adopt the same design rules as in [18] to design the K sets of dispersion matrices of the proposed K-STBCs. Equation (6) can be reformulated as

$$\boldsymbol{x}_k = \bar{\boldsymbol{A}}_k \boldsymbol{s}_k, \tag{10}$$

where $\boldsymbol{s}_k \in \mathbb{C}^{P_k \times 1}$ is the kth symbol vector. Using (8) and (10), we can express (5) as

$$\boldsymbol{x} = \boldsymbol{P}\left[\left(\bar{\boldsymbol{A}}_1 \boldsymbol{s}_1\right) \otimes \cdots \otimes \left(\bar{\boldsymbol{A}}_K \boldsymbol{s}_K\right)\right]. \tag{11}$$

Using the property $(\boldsymbol{A} \otimes \boldsymbol{B})(\boldsymbol{C} \otimes \boldsymbol{D}) = \boldsymbol{AC} \otimes \boldsymbol{BD}$ of Kronecker product leads to

$$\mathrm{vec}(\boldsymbol{X}) = \boldsymbol{P}\big(\bar{\boldsymbol{A}}_1 \otimes \cdots \otimes \bar{\boldsymbol{A}}_K\big)\underbrace{(\boldsymbol{s}_1 \otimes \cdots \otimes \boldsymbol{s}_K)}_{\boldsymbol{s}}, \quad (12)$$

where $\boldsymbol{s}$ is the transmitted coded vector having dimensions $P = P_1 \ldots P_K$. The input-output relation in (1) can also be rewritten by using the property $\mathrm{vec}(\boldsymbol{ABC}) = (\boldsymbol{C}^{\mathrm{T}} \otimes \boldsymbol{A})\,\mathrm{vec}(\boldsymbol{B})$ as

$$\boldsymbol{y} = \Big(\boldsymbol{h}^{\mathrm{T}} \otimes \boldsymbol{I}_N\Big)\boldsymbol{x} + \boldsymbol{n}, \quad (13)$$

which can be expanded using (11) as

$$\boldsymbol{y} = \Big(\boldsymbol{h}^{\mathrm{T}} \otimes \boldsymbol{I}_N\Big)\boldsymbol{P}\left(\underbrace{\otimes_{k=1}^{K}\bar{\boldsymbol{A}}_k}_{NM\times P}\right)\underbrace{\boldsymbol{s}}_{P\times 1} + \boldsymbol{n}. \quad (14)$$

Finally, the above equation can be compactly written as

$$\boldsymbol{y} = \bar{\boldsymbol{H}}\boldsymbol{s} + \boldsymbol{n}, \quad (15)$$

where $\bar{\boldsymbol{H}} = (\boldsymbol{h}^{\mathrm{T}} \otimes \boldsymbol{I}_N)\boldsymbol{P}(\otimes_{k=1}^{K}\bar{\boldsymbol{A}}_k) \in \mathbb{C}^{N\times P}$ is the *equivalent channel matrix*, and $\boldsymbol{s} \in \mathbb{C}^{P\times 1}$ is the Kronecker-encoded symbol vector, given by the Kronecker product of the *K* input symbol vectors. Note that, the equivalent channel $\bar{\boldsymbol{H}}$, of the K-STBCs scheme involves the Kronecker product of the effective LD code matrices $\bar{\boldsymbol{A}}_1, \ldots, \bar{\boldsymbol{A}}_K$. The rate *R*, i.e., symbols/transmission of the K-STBCs is given by

$$R = \frac{\sum_{k=1}^{K} P_k}{\Pi_{k=1}^{K} P_k}\log_2 \mu, \quad (16)$$

where μ is the modulation cardinality, and P_k is the length of the symbol vector involved in the design of K-STBCs.

A. Kronecker of Multiple Alamouti STBCs as an Example

In the special case of a real-valued Alamouti design [16], we consider the two dispersion matrices given as

$$\boldsymbol{A}_1^{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{A}_2^{(1)} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \quad (17)$$

Constructing the LD code matrix $\bar{\boldsymbol{A}}_k$ from the dispersion matrices as explained in (8) leads to

$$\bar{\boldsymbol{A}}_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}. \quad (18)$$

According to (18), we notice that there is only one non-zero entry at each row, which fulfills the design requirement of the LD code matrix $\bar{\boldsymbol{A}}_1$. In this case, the equivalent channel $\bar{\boldsymbol{H}}$ is the Kronecker product between two Alamouti codes, i.e., $(\boldsymbol{h}^{\mathrm{T}} \otimes \boldsymbol{I}_N)\boldsymbol{P}(\bar{\boldsymbol{A}}_1 \otimes \bar{\boldsymbol{A}}_2)$ and assuming $\bar{\boldsymbol{A}}_2 = \bar{\boldsymbol{A}}_1$ yields

$$\bar{\boldsymbol{H}} = \left[\begin{array}{cc|cc} h_1 & h_2 & h_3 & h_4 \\ h_2 & -h_1 & h_4 & -h_3 \\ \hline h_3 & h_4 & -h_1 & -h_2 \\ h_4 & -h_3 & -h_2 & h_1 \end{array}\right]. \quad (19)$$

Note that the equivalent channel $\bar{\boldsymbol{H}}$ is quasi-orthogonal.

Remark: By inspecting (18), note that the LD code has a single non-zero element per row, following the same constraint as the classical LD code [18].

III. Receiver Design

The maximum-likelihood (ML) receiver operates with the following minimum distance rule

$$\hat{\boldsymbol{s}} = \arg\min_{\boldsymbol{s}}\big\|\boldsymbol{y} - \bar{\boldsymbol{H}}\boldsymbol{s}\big\|_2^2. \quad (20)$$

Assuming additive white Gaussian noise (AWGN), the classical ZF receiver can achieve the ML estimate. The ZF and MMSE filter $\boldsymbol{W}$ is designed from the equivalent virtual channel matrix $\bar{\boldsymbol{H}}$. The ZF equalizer is given as

$$\boldsymbol{W} = \Big(\bar{\boldsymbol{H}}^{\mathrm{H}}\bar{\boldsymbol{H}}\Big)^{-1}\bar{\boldsymbol{H}}^{\mathrm{H}}, \quad (21)$$

and the MMSE equalizer is given as

$$\boldsymbol{W} = \Big(\bar{\boldsymbol{H}}^{\mathrm{H}}\bar{\boldsymbol{H}} + \sigma_n^2\boldsymbol{I}_M\Big)^{-1}\bar{\boldsymbol{H}}^{\mathrm{H}}. \quad (22)$$

The Kronecker-encoded vector $\hat{\boldsymbol{s}}$ can be estimated according to the ZF / MMSE solution, yielding

$$\hat{\boldsymbol{s}} = \boldsymbol{W}\boldsymbol{y} = \boldsymbol{W}\bar{\boldsymbol{H}}\boldsymbol{s} + \boldsymbol{W}\boldsymbol{n}. \quad (23)$$

The decoupled estimates of the *K* symbol vectors are obtained by solving:

$$\min_{\boldsymbol{s}_1 \ldots \boldsymbol{s}_K}\|\hat{\boldsymbol{s}} - \boldsymbol{s}_1 \otimes \cdots \otimes \boldsymbol{s}_K\|_2^2. \quad (24)$$

The key fact is that (24) can be recast as a rank-one *K*th order tensor as $\mathcal{S} = \boldsymbol{s}_K \circ \cdots \circ \boldsymbol{s}_1 \in \mathbb{R}^{P_K \times P_{K-1} \times \cdots \times P_1}$, yielding the following tensor rank-one approximation problem

$$\min_{\boldsymbol{s}_1,\ldots,\boldsymbol{s}_K}\Big\|\hat{\mathcal{S}} - \boldsymbol{s}_K \circ \cdots \circ \boldsymbol{s}_1\Big\|_{\mathrm{F}}^2, \quad (25)$$

where $\hat{\mathcal{S}} \in \mathbb{C}^{P_K \times P_{K-1} \times \cdots \times P_1}$ is the *K*-th order "tensorized" received data tensor. Note that the estimated tensor $\hat{\mathcal{S}}$ is a scaled version of the original tensor $\mathcal{S}$ combined with the channel $\bar{\boldsymbol{H}}$ and corrupted by AWGN. Therefore, estimating the transmitted symbols $\boldsymbol{s}_K, \ldots, \boldsymbol{s}_1$ can be translated into finding a rank-one approximation to $\hat{\mathcal{S}}$. A solution to this problem can be found from the higher-order singular value decomposition (HOSVD) [23], [24]. In this letter, we resort to the tensor-power-method detector (TPMD) originally proposed in [19] as an efficient way to solve the problem (25). For simplicity, let us define the following Gramian

$$\boldsymbol{B}_k = \mathbb{E}\Big[[\hat{\mathcal{S}}]_{(k)}[\hat{\mathcal{S}}]_{(k)}^{\mathrm{H}}\Big] \in \mathbb{C}^{P_k \times P_k}, \quad (26)$$

where $[\hat{\mathcal{S}}]_{(k)}$ is the *k*-mode unfolding of the received tensor $\hat{\mathcal{S}}$, which can be expanded as

$$[\hat{\mathcal{S}}]_{(k)} = \big(\boldsymbol{W}\bar{\boldsymbol{H}}\big)\boldsymbol{s}_k\Big(\otimes_{i\neq k}\boldsymbol{s}_i^{\mathrm{T}}\Big) + [\mathcal{N}]_{(k)} \quad (27)$$

where $\mathcal{N} \in \mathbb{C}^{P_K \times P_{K-1} \times \cdots \times P_1}$ is the scaled noise tensor. This can be further written as

$$[\hat{\mathcal{S}}]_{(k)} = \boldsymbol{s}_k\Big(\otimes_{i\neq k}\boldsymbol{s}_i^{\mathrm{T}}\Big) + [\mathcal{N}]_{(k)}. \quad (28)$$

Therefore, (26) can be written as

$$\boldsymbol{B}_k = \mathbb{E}\Big[\Big(\boldsymbol{s}_k\Big(\otimes_{i\neq k}\boldsymbol{s}_i^{\mathrm{T}}\Big) + [\mathcal{N}]_{(k)}\Big) \Big(\boldsymbol{s}_k\Big(\otimes_{i\neq k}\boldsymbol{s}_i^{\mathrm{T}}\Big) + [\mathcal{N}]_{(k)}\Big)^{\mathrm{H}}\Big]. \quad (29)$$

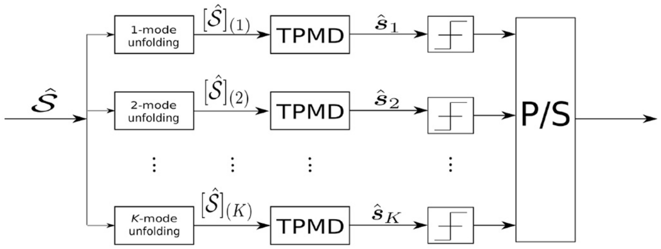

Fig. 1. K-STBCs decoder as parallel TPMD branches.

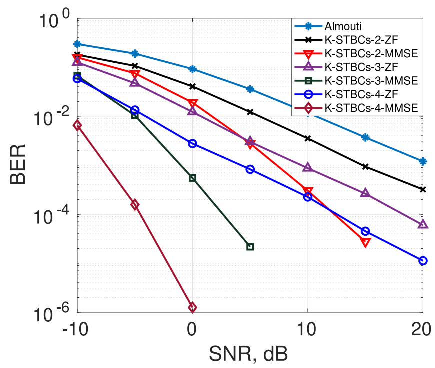

Fig. 2. Performance comparison of different K-STBCs by varying the number of transmit antennas and their corresponding code rate.

As the signal and noise terms are uncorrelated, by applying the expectation operator $\mathbb{E}$, we get

$$\boldsymbol{B}_k = \boldsymbol{s}_k \underbrace{\left(\otimes_{i\neq k}\boldsymbol{s}_i^{\mathrm{T}}\right)\left(\otimes_{i\neq k}\boldsymbol{s}_i^{*}\right)}_{\gamma} \boldsymbol{s}_k^{\mathrm{H}} + \sigma_n^2 \boldsymbol{I}_{P_k}, \quad (30)$$

$$\boldsymbol{B}_k = \gamma \boldsymbol{s}_k \boldsymbol{s}_k^{\mathrm{H}} + \sigma_n^2 \boldsymbol{I}_{P_k}. \quad (31)$$

Since $\boldsymbol{s}_k\boldsymbol{s}_k^{\mathrm{H}}$ is a rank-one matrix, we can resort to the TPMD algorithm of [19] to find the estimate $\hat{\boldsymbol{s}}_k, k = \{1,\ldots,K\}$ as the dominant left singular vectors of the k-mode unfolding $[\hat{\mathcal{S}}]_{(k)}$ of the tensor $\hat{\mathcal{S}}, \{k = 1,\ldots,K\}$. The decoding of the k-th symbol vcector follows by projecting $\hat{\boldsymbol{s}}_k$ onto the known finite alphabet.

The decoding scheme of the proposed K-STBC is shown in Fig. 1, where the number K of TPMD branches corresponds to K symbol vectors $\boldsymbol{s}_k$, involved in the construction of K-STBCs. The estimation of the K transmitted symbol vectors is executed in parallel, since the K decoding branches are independent. The overall complexity of the decoder after the equalizer is then given as $\mathcal{O}(8KJP_k^K)$, where J is the number of iterations necessary for convergence [19].

IV. Simulation Results

For simplicity, we consider the special case of an Alamouti scheme to evaluate the performance of the proposed K-STBCs, although the proposed framework also includes any type of LD codes [18]. We consider a system with M = 2, 4, 8, 16 transmit antennas operating with BPSK, QPSK, and 16-PSK.

Fig. 2 shows the performance of different K-STBCs schemes combined with the ZF or MMSE equalization. Note that the BER performance improves when the number of component Alamouti STBCs involved in the composition of the K-STBCs is increased. However, such an improved performance comes at the cost of a reduced data rate. Considering the Alamouti scheme with $M = 2$ and our proposed scheme with $M = 4$, both schemes have rate $R = 1$ symbols/transmission, but the proposed Kronecker coding scheme achieves a better performance, thanks to the introduction of the rank-one detector which exploits the rank-one tensor structure of the transmitted data symbols, yielding an additional noise rejection gain. Therefore, K-STBCs-ZF with $M = 4$ shows a 6 dB performance improvement over the Alamouti scheme at a BER of 10^{-2}. Similarly, by increasing the number of Kronecker products among the LD matrices leads to an improved BER performance, at the cost of a lower code rate as shown in Table I. Similarly, for a target BER of 10^{-2}, K-STBCs-4-ZF with $M = 16$ provides approximately a 10 dB gain compared to K-STBCs-2-ZF at the cost of a lower transmission rate.

TABLE I
Code Rates of the Different K-STBC Schemes

Transmit antennas	Virtual MIMO channel	Rate
2	2×2	1
4	4×4	1
8	8×8	3/4
16	16×16	1/2

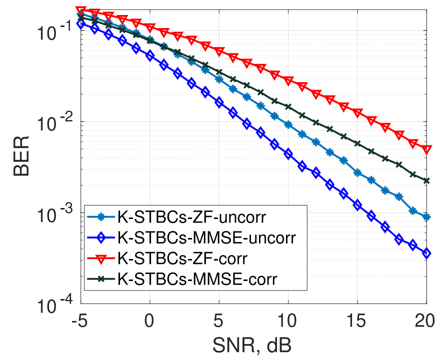

Fig. 3. Performance comparison of correlated and uncorrelated Rayleigh fading channels for K-STBCs approach using QPSK.

The performance comparison of the proposed scheme over correlated [21] and uncorrelated channels both for the ZF and MMSE equalizers using QPSK is shown in Fig. 3. The result demonstrates that channel correlation has a significant impact on the performance.

Fig. 4 and Fig. 5 depicts a performance comparison of the proposed K-STBCs using ZF and MMSE equalization with the existing quasi-orthogonal STBCs such as ABBA [15] and EA STBCs [16], [17] considering the same number of transmit antennas ($M = 4$), spectral efficiency and unitary transmission rate. The performance of K-STBCs is further compared with the theoretical bound, i.e., the Gaussian approximation [22]. It can be seen that K-STBCs outperform ABBA and EA STBCs. Such a performance gain comes from the inherent Kronecker coding gain introduced by the mutual spreading of multiple space-time codewords, which is extracted by a simple tensor rank-one approximation algorithm that solves problem (25). Such a gain is not achieved by the ABBA and EA STBCs decoding algorithms. Note that, if a lower constellation order is used, for instance, BPSK, then the MMSE equalizer shows an improvement in both scenarios, i.e., with and without using

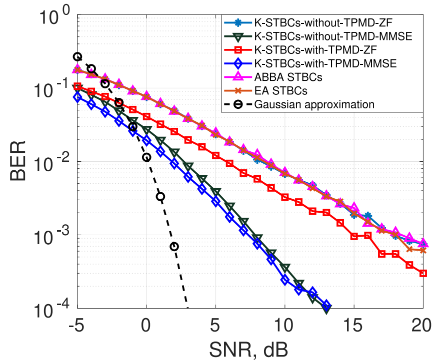

Fig. 4. Performance comparison of K-STBCs using BPSK constellation for uncorrelated fading channels.

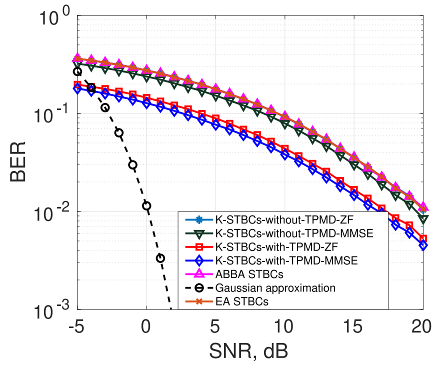

Fig. 5. Performance comparison of K-STBCs using 16-PSK constellation for uncorrelated fading channels.

the TPMD receiver. This is due to the fact that the *a priori* knowledge of the noise variance significantly helps the symbol detection processes and secondly, BPSK has a maximum Euclidean distance between the two constellation points which facilitates the hard decision. When considering higher-order constellations, MMSE equalization is not good enough, and, therefore, the additional impact of TPMD becomes dominant due to the additional noise rejection gain, thus helping in the hard decision. A performance improvement can only be achieved with the TPMD receiver, as shown in Fig. 5. Please note that in the low SNR regime, the K-STBCs designs achieve a lower BER than the Gaussian approximation bound [22]. The reason behind is that the Gaussian approximation assumes i.i.d. Gaussian distributed symbols while the proposed scheme introduces a controlled correlation on the transmitted symbols that is efficiently exploited by the TPMD receiver to achieve higher coding gains and an enhanced noise rejection.

V. Conclusion

Kronecker product space-time block codes (K-STBCs) are based on the mutual spreading of multiple codeword matrices. The proposed K-STBCs inherit the so-called Kronecker coding of the involved transmitted symbols vectors. By rearranging the received data after space-time filtering as a higher-order tensor, the mutually-encoded symbol vectors can be decoded by means of a rank-one tensor power method detection algorithm. The proposed receiver achieves an efficient noise rejection and yields an improved bit-error-ratio performance, especially in the low SNR regime, in comparison to competing codes such as the ABBA and EA-STBCs.

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